Hybrid stars using the quark-meson coupling and proper-time Nambu–Jona-Lasinio models

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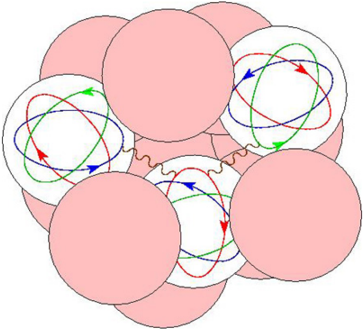
Overview



Neutron Stars

A few fast facts

- Neutron stars are quite complex objects and a complete understanding has not yet been achieved.
- Their densities are far greater than what we are capable of maintaining in a laboratory. They are the densest forms of matter this side of an event horizon !
- All 4 forces play a role in neutron stars.
- An ideal place to test phenomenological nuclear physics models in an extreme environment.

Typical properties of Neutron Stars

- $M \sim 1.4 - 3.0 M_{\odot}$
- $R \sim 10$km
- $\rho \sim 1 - 10\rho_0$, $\rho_0 \sim 0.16$fm^{-3}.
- $\vec{B} \sim 10^9$–10^{15}G, most $\vec{B} \sim 10^{12}$G
- Period of rotation $P \sim 1.6$ ms – 4.6s
- $T \sim 10^{11}$–10^{12}K for new, $T \sim 10^6$K for old, but usually assume $T = 0$ for old NS as $kT \ll E_F$

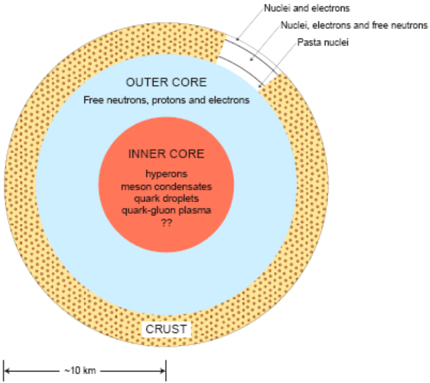
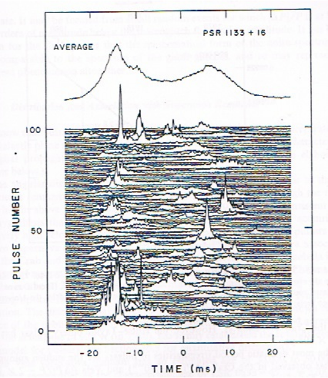


FIGURE : A schematic picture of a neutron star (G. Baym).

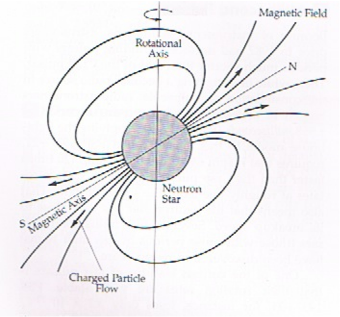
A Neutron Star



Refs. [Shapiro, (1983)], [Zeilik, (1998)] .

Key observational facts :

- Periods ∼ms – s
- Periods increase very slowly and
- pulsars are very good clocks.



Corpses of a Main Sequence Star

Eventually the fuel becomes depleted and the outward pressure can no longer prevent the gravitational collapse of the star. The star begins to contract and depending on the initial mass of the star, a number of final states (or corpses) can be obtained such as

- White dwarf ($M < 1.4 M_{\odot}$)- Progenitor $M \leq 7.0 M_{\odot}$
- Neutron star ($1.4 \leq M \leq 3.0 M_{\odot}$) - Progenitor $M \sim 8 - 20 M_{\odot}$ (Right - The Crab nebula, NS at center)
- Black hole ($M > 3.0 M_{\odot}$)

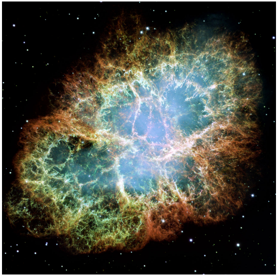


FIGURE : Crab nebula.

WD + NS are held up by degeneracy pressure, whereas black holes are a victory for gravity.

Hydostatic Equilibrium - Newtonian Gravity

Hydrostatic equilibrium in a white dwarf can be described quite adequately using Newtonian physics and the equations of structure for such stars are given by the following coupled differential equations :

$$\frac{dp}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

$$\rho(r) = \frac{\epsilon}{c^2}$$

Hydrostatic Equilibrium - GR Correction

The above structure equations are only suitable if the mass of the star under consideration is not large enough to significantly warp space-time. General relativistic effects become important when the ratio

$$\frac{GM}{c^2 R} \gtrsim 0.1$$

For neutron stars it is necessary to include GR effects . The spherically symmetric static line element is

$$ds^2 = e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) - e^{\nu(r)} dt^2$$

Einstein's Eqn.'s $\rightarrow$ GR Hydrostatic Equilibrium

Hydrostatic Equilibrium - GR Correction

For a static spherically symmetric non-rotating star the pressure gradient is now given by

$$\frac{dP}{dr} = -(\rho(r) + P(r))\frac{M(r) + 4\pi r^3 P(r)}{r^2(1 - \frac{2M(r)}{r})}$$

$$= -\frac{G\epsilon(r)M(r)}{(cr)^2}(1 + \frac{P(r)}{\epsilon(r)})(1 + \frac{4\pi r^3 P(r)}{M(r)c^2})(1 - \frac{2GM(r)}{c^2 r})^{-1}$$

whereas the second equation remains unaltered.

As can be seen in the second equality the terms that modify the pressure gradient are all positive definite and greater than 1

The First Calculation of NS Models with a simple EoS

The first numerical calculation was done by Oppenheimer and his student Volkoff.
Assumed matter to be an ideal gas of free neutrons.

$$M_{\mathrm{max}} = 0.7 M_{\odot}$$

$$R = 9.6\text{km}$$

$$\rho_c = 5 \times 10^{15}\text{gcm}^{-3}$$

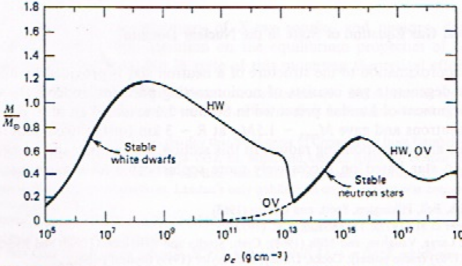


Figure 9.1 Gravitational mass versus central density for the HW (1958) and OV (1939) equations of state. The stable white dwarf and neutron star branches of the HW curve are designated by a *heavy solid* line.

NO interaction, missing the strong interaction which is very STRONG !
Unable describe neutron stars $M \sim 1.4 M_{\odot}$!

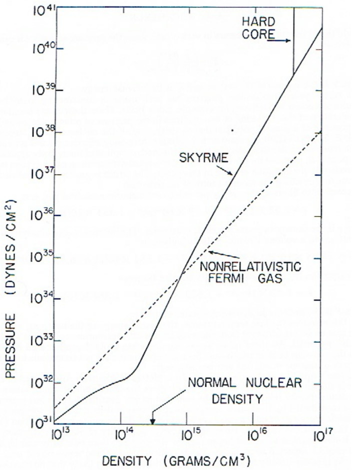
Strong Interaction and Neutron Star Models

Cameron (1959) showed using a Skyrme force that nuclear forces significantly stiffen EoS.

$$M_{max}^{\rm Skyrme} \sim 2.0 M_\odot \gg M_{\rm max}^{\rm ideal} \sim 0.7 M_\odot$$

Which means these compact stars can be formed in a supernova as suggested by Baade and Zwicky in 1934.

"With all reserve we advance the view that supernovae represent the transitions from ordinary stars into neutron stars, which in their final stages consist of extremely closely packed neutrons."



Which Model ?

For a realistic model, it would be ideal to derive it directly from QCD. This unfortunately does not seem feasible at the moment without some new deep physical insight simplifying the QCD equations of motion.

Many models available, but which one ?

Try them all and filter them based on general properties :

- Relativity
- Particle content
- Nuclear matter observables
- Finite nuclei and hypernuclei observables
- and astrophysical observables

Filtering ? ?

Relativity

- Lorentz covariance is a fundamental symmetry.
- GR effects are important for neutron stars

$$\frac{GM}{c^2 R} \sim 0.2$$

- It is known that there are large scalar and vector potentials
- No superluminal speed of sound in matter, Speed of Sound $<$ Speed of Light

Nuclear Matter Observables

- Incompressibility
- Symmetry energy and its slope,...

Composition

- The only possible constituents of dense matter in thermodynamic and β - equilibrium are the particles which cannot decay or escape.
- The composition of the neutron stars core beyond N and e is uncertain.
- Possibly contain heavier particles such as $\mu, \pi^-, \bar{K}^-, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}$ and/or deconfined u, d and s quarks. We will not consider meson condensates.

Finite Nuclei and Hypernuclei Observables

- Ground and excited state properties
- Static and dynamic properties
- density distributions, single particle energies, GRs, pygmy resonances ....
- fission barriers, ...

Composition and the EoS

The inclusion of additional degrees of freedom beyond n, p and e will result in softening of the EoS.

Softening of the EoS results in a smaller maximum mass of the star.

Historically masses of neutron stars have been found to be around $M \sim 1.4M_{\odot}$, but recently there have been observations of 2 high mass neutron stars of $M \sim 2.0M_{\odot}$.

High Mass Neutron Stars $\Longleftrightarrow$ Stringent Constraint on EoS at High Density

[Demorest *et al*, (2010)] PSR J1614-2230 $M = 1.97 \pm 0.04M_{\odot}$	[Antoniadis *et al*, (2013)] PSR J0348+0432 $M = 2.01 \pm 0.04M_{\odot}$

Which has led to many models incorporating exoctica, such as hyperons and quarks, to be ruled out due to these observations

Exotica and the EoS

Hyperons must be considered

- because if the system survives longer than the time scales of the weak interactions 10^{-9}secs it is able to reach equilibrium w.r.t. beta decay.
- Also, as the density increases so does the chemical potentials of the nucleons. Eventually it will become energetically favorable to turn nucleons at the top of the Fermi sea into hyperons.
- Phenomenologically Σ–A interaction is repulsive. Explained very naturally within the QMC model [Guichon *et al*, (2008)] to be a result of one gluon exchange.
- This is not the case for Λ hypernuclei

$$(M_\Lambda - M_N) \sim 170 \text{ MeV and } (M_\Xi - M_N) \sim 380 \text{ MeV}$$

In β-equilibrium Ξ^- and Λ compete for dominance. This is possible since $\mu_{\Xi^-} = \mu_n + \mu_e$, where $\mu_e \sim 200$ MeV

What we want is to solve for the number densities of the particles of interest in β-equilibrium with two constraints : charge neutrality and baryon number conservation.

Neutrinos and photons are generally taken to radiate away from the neutron star.

The system of equations that needs to be solved for the spin – $\frac{1}{2}$ octet in generalised β-equilibrium under the constraints of charge neutrality and baryon number conservation is

$$
\begin{aligned}
0 &= \mu_i + B_i\lambda + \nu Q_i \ , \\
0 &= \mu_\ell - \nu \ , \\
0 &= \sum_i B_i\rho_i - \rho \ , \\
0 &= \sum_i B_i\rho_i Q_i + \sum_\ell \rho_\ell Q_\ell \ ,
\end{aligned}
$$

$i \in \{N, Y, \ell\}$, ρ_i = densities, B_i = baryon number, Q_i = charge and the Lagrange multipliers ν, λ.

Traditional approach

Fit many parameters $\sim 20 - 30$ parmeters to N – N scattering data and further parameters fit to light nuclei to be able to describe the 3 – body force.

Cannot do this with hyperons – Not enough data !
This suggest an alternative approach.

Use an effective model, here we will discuss the application of a relativistic quark level model, the QMC model.

Quark-Meson Coupling Model

Quarks couple to mesons

Original idea due to Guichon [Guichon, (1988)]

Improved and generalised to finite nuclei by Guichon, Rodinov and Thomas [Guichon *et al*, (1996)]. A lot of work has been done on developing the QMC model [Saito *et al*, (2007)].

Starts with a quark model (MIT bag) and introduces a relativistic Lagrangian with mesons coupling to quarks.

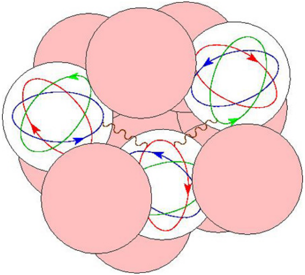


FIGURE : QMC Model (Guichon)

Meson couplings determined by saturation properties of nuclear matter :

$$g_\sigma^q,\, g_\omega^q\,,\, g_\rho^q \iff \rho_0,\, \mathcal{E}_0,\, a_{\mathrm{sym}}$$

Strong Interaction

Considered through the exchange of massive mesons :

- σ scalar-isoscalar (attractive)
- ω vector-isoscalar (repulsive)
- ρ vector-isovector (isospin dependent)
- π (Only contributes through its Fock term due to parity considerations)

Only couple to the light quarks.

Could be supplemented with heavy mesons δ, K^*, σ^*, ϕ, . . . of course more model dependent

Bag Model Parameterisation

Solve self-consistently for the internal structure and parameterise as a function of the mean scalar field

$$M_B^* = M_B - w_{\sigma B}\, g_{\sigma N}\, \bar{\sigma} + \frac{d}{2} \tilde{w}_{\sigma B} \left(g_{\sigma N} \bar{\sigma} \right)^2$$

We can solve for the EoS at the hadronic level (Walecka/QHD Models).
Sub-structure of the baryons are entirely contained in the mass parameterisation.

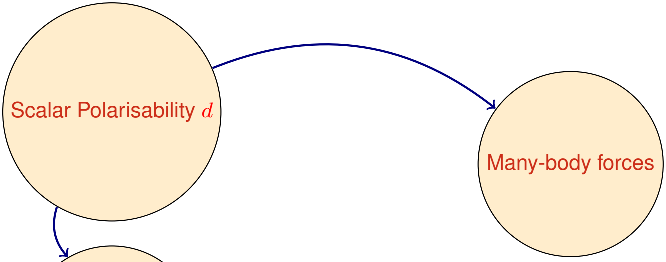
The mass parameterisation includes gluon exchange

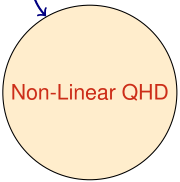
$$M(R) = \sum_i \frac{\Omega_i - Z_i}{R} + \frac{4\pi}{3} R^3 B + \Delta E_g^M$$

$$\Delta E_g^M = -\frac{\alpha_s}{2} \sum_{i<j} \int d^3x\, \vec{B}_i^a \cdot \vec{B}_j^a$$

Λ–Σ mass splitting,
Unbound Σ–hypernuclei
[Guichon *et al*, (2008)]

Scalar Polarisability d





- Can re-write non-linear coupling as a linear coupling and non-linear self-coupling
- Scalar polarisability possible physical origin of NL versions of Quantum Hadro-Dynamics (QHD, Baryons pt particles)

- (Guichon and Thomas,2004) derived a Skyrme type effective force with 2, 3 and 4-body forces from the QMC model.
- Improved in (Guichon *et al*, 2006)
- d was the key ingredient, many-body terms proportional to powers of d
- Many-body interactions were a direct consequence of nucleon substructure

The Devil is in the Details

The Hadronic Lagrangian

The QMC Lagrangian density used in this work is given by a combination of baryon, meson, and lepton components

$$\mathcal{L} = \sum_B \mathcal{L}_B + \sum_m \mathcal{L}_m + \sum_\ell \mathcal{L}_\ell \ ,$$

for the octet of baryons $B \in \{N, \Lambda, \Sigma, \Xi\}$, selected mesons $m \in \{\sigma, \omega, \rho, \pi\}$, and leptons $\ell \in \{e^-, \mu^-\}$ densities.

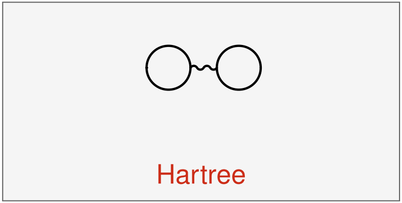
$$\begin{aligned} \Gamma_{\sigma B} &= g_{\sigma B} C_B(\bar{\sigma}) F^\sigma(k^2)\mathbf{1} \\ &= -\frac{\partial M_B^*}{\partial \bar{\sigma}} F^\sigma(k^2)\mathbf{1} \ , \\ \mathbf{\Gamma}_{\eta B} &= \epsilon_\eta^\mu \mathbf{\Gamma}_{\mu\eta B} \\ &= \epsilon_\eta^\mu \left[g_{\eta B}\gamma_\mu F_1^\eta(k^2) + \frac{i f_{\eta B}\sigma_{\mu\nu}}{2M_B^*} k^\nu F_2^\eta(k^2) \right] \boldsymbol{t} \ ; \\ & \quad \eta \in \{\omega, \rho\} \ , \\ \mathbf{\Gamma}_{\pi B} &= i\frac{g_A}{2f_\pi} F^\pi(k^2)\gamma^\mu k_\mu \gamma_5 \boldsymbol{\tau} \ , \end{aligned}$$

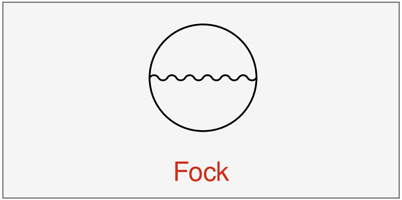
Summary of Approximations

E.O.M. are a system of coupled equations that are very difficult to solve. Need to make approximations to make the problem tractable !

- Consider nuclear matter (infinite and homogeneous, No Coulomb interaction nor Surface effects)
- No Dirac sea
- Mean Field Approximation (Hartree or Hartree-Fock Approximation)
- Zero temperature
- Time independence (Static Approximation)
- Form factors (Dipole) and contact subtraction ($\delta(\vec{r}) \mapsto \xi \times \delta(\vec{r})$, $\xi = 0$) in Fock terms

Hartree and Hartree-Fock MFA





In the Hartree MFA,

$$\alpha \rightarrow \langle \alpha \rangle \ ,$$

where $\alpha \in \{\sigma, \omega, \rho\}$.

To go beyond the Hartree MFA, we follow the method of (Guichon,2006) and introduce a fluctuation part in addition to the mean field :

$$\alpha \rightarrow \langle \alpha \rangle + \delta\alpha$$

$$\begin{aligned} \bar{\Psi}\Gamma_\alpha\Psi &= \langle \bar{\Psi}\Gamma_\alpha\Psi \rangle + \left(\bar{\Psi}\Gamma_\alpha\Psi - \langle \bar{\Psi}\Gamma_\alpha\Psi \rangle \right) \\ &= \langle \bar{\Psi}\Gamma_\alpha\Psi \rangle + \delta\left(\bar{\Psi}\Gamma_\alpha\Psi \right) \end{aligned}$$

and solve the E.O.M. order by order.

Symmetric Nuclear Matter Properties

Symmetric Nuclear Matter Properties

Model/ Scenario	$g_{\sigma N}$	$g_{\omega N}$	g_ρ	K_0 [MeV]	L_0 [MeV]	U_Λ [MeV]	U_{Σ^-} [MeV]	U_{Ξ^-} [MeV]
Standard	8.97	9.38	4.96	273	84	3	26	5
$\Lambda = 1.3$	9.31	10.67	5.40	289	88	29	53	18
Dirac Only	10.10	9.22	7.84	294	85	-23	4	-8
Hartree Only	10.25	7.95	8.40	283	88	-49	-23	-21
$\Lambda = 1.1, g_{\sigma Y} \times 1.3$	9.16	10.06	5.16	283	86	-15	14	-4
$\Lambda = 2.0, g_{\sigma Y} \times 1.9$	9.69	12.27	6.16	302	92	-29	20	-7

Standard (Baseline) scenario : $\Lambda = 0.9$GeV , tensor couplings obtained from exp. magnetic moments ($\kappa_{\rho N} = f_{\rho N}/g_{\rho N} = 3.7$, equivalent to bag model)

$$K_0 = 9 \left(\frac{\partial P}{\partial \rho} \right)_{\rho=\rho_0}$$

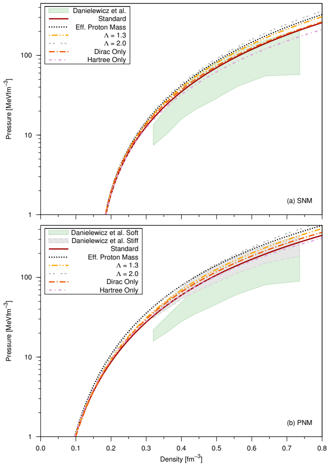
J. R. Stone (2013)
$250 \leq K_0 \leq 315$ MeV

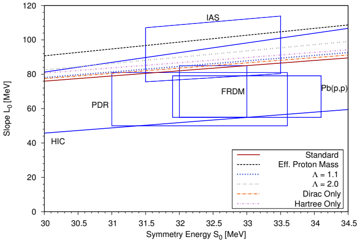
Slope of symmetry energy at ρ_0

$$L_0 = L(\rho_0) = 3\rho_0 \left(\frac{\partial S}{\partial \rho} \right)_{\rho=\rho_0}$$

RMF models generally predict larger values of L_0, E.g. QHD – NL3, NL-SH, NLC, TM1, TM2 $L_0 \sim 110$–120MeV

Nuclear Matter Properties



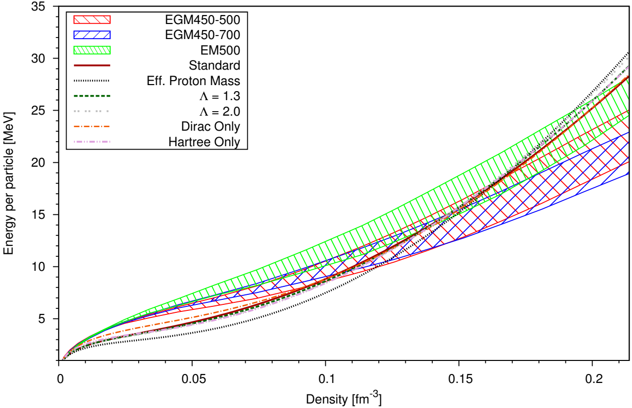


Linear Correlation Between S Vs L. Also seen in Quantum Monte Carlo [Gandolfi *et al*, (2012)] and CEFT [Tsang *et al*, (2012)] calculations.

$$S(\rho) = \frac{1}{2}\left(\frac{\partial^2(\frac{\epsilon}{\rho})}{\partial\beta^2}\right)_{\beta=0}, \; S_0 \equiv S(\rho_0)$$

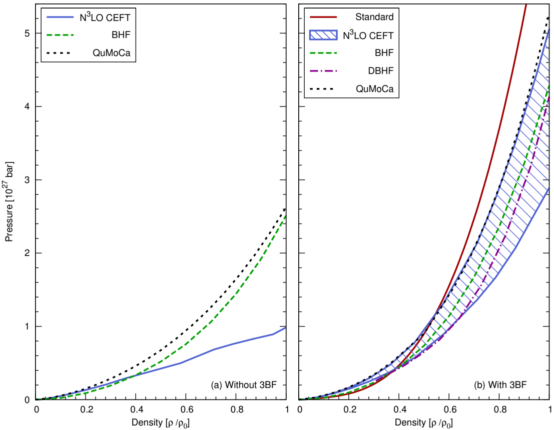
$$L(\rho) = 3\rho\left(\frac{\partial S}{\partial \rho}\right), \; L_0 \equiv L(\rho_0)$$

PNM : QMC/CEFT Comparison



Comparison to CEFT at N^3LO (Tews *et al*, 2013)

PNM : QMC Comparison 2 and 3 Body Interactions

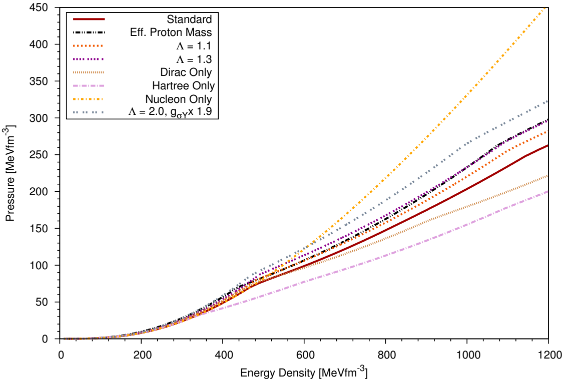


QMC naturally incorporates many-body forces

(Left) 2-Body
(Right) 3-Body

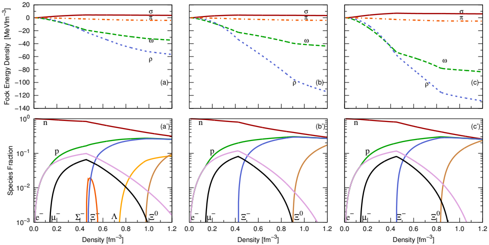
β–Equilibrium – Neutron Star Matter

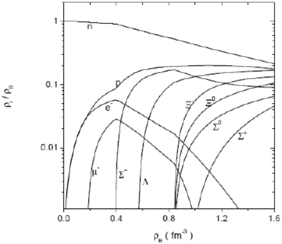
Fock terms stiffen EoS
and
Hyperons soften it

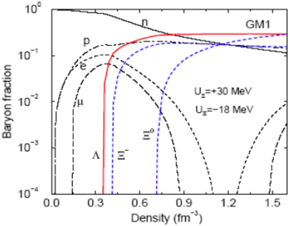


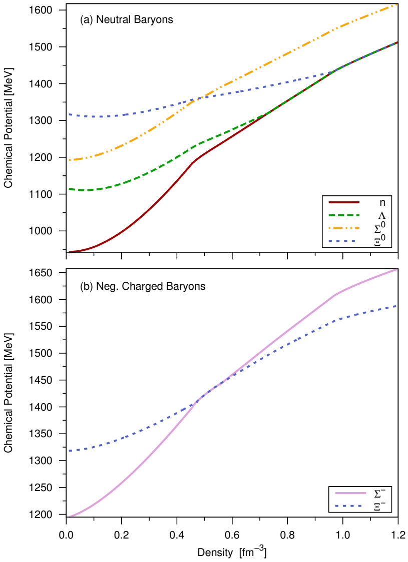
Hartree → Dirac Only → Full Fock Terms

Different composition from other models - E.g. QHD (Schaffner-Beliech, 2005)

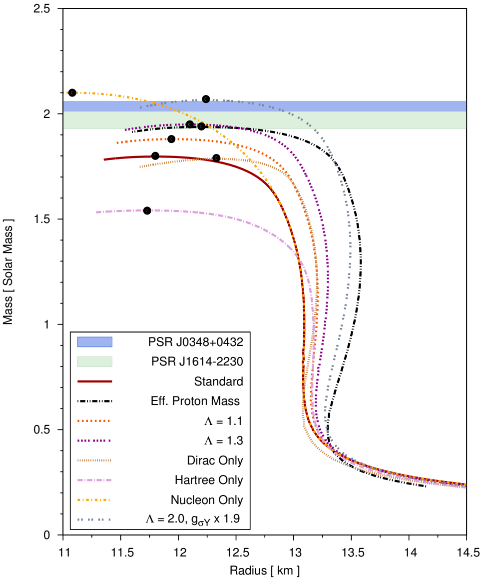








(a) Neutral Baryons
Chemical Potential [MeV]
1600
1500
1400
1300
1200
1100
1000
n
Λ
Σ^0
Ξ^0
(b) Neg. Charged Baryons
Chemical Potential [MeV]
1650
1600
1550
1500
1450
1400
1350
1300
1250
1200
Σ^-
Ξ^-
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Density [fm^{-3}]



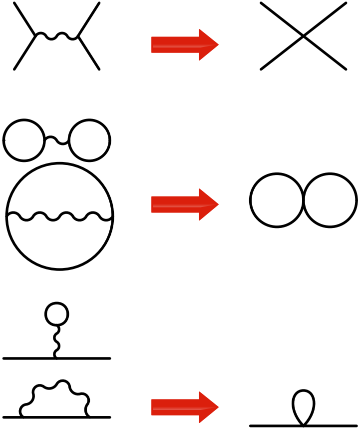
Neutron Star Properties

Model/ Scenario	$M_{\max}$ $[M_\odot]$	R [km]	$\rho_c^{\max}$ $[\rho_0]$
Standard	1.80	11.80	5.88
$\Lambda = 1.3$	1.95	12.10	5.52
Dirac Only	1.79	12.33	5.22
Hartree Only	1.54	11.73	6.04
Nucleon Only	2.10	11.08	6.46
$\Lambda = 1.1, g_{\sigma Y} \times 1.3$	1.84	11.91	5.78
$\Lambda = 2.0, g_{\sigma Y} \times 1.9$	2.07	12.24	5.38

Nambu-Jona-Lasinio Model

Lagrangian and Effective Potential

4 Fermi Contact Interaction Between Quarks



$$
\begin{aligned}
\mathcal{L}_{\rm NJL} &= \bar{\psi}(i\partial\!\!\!/ - \hat{m}_0)\psi \\
&+ G_{\rm S} \sum_{a=0}^{N_{\rm F}^2-1} \left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_a\psi\right)^2 \right] \\
&- G_{\rm V} \sum_{a=0}^{N_{\rm F}^2-1} \left[\left(\bar{\psi}\gamma_\mu\lambda_a\psi\right)^2 + \left(\bar{\psi}\gamma_\mu\gamma_5\lambda_a\psi\right)^2 \right],
\end{aligned}
$$

Schwinger's Proper Time Regularisation

The NJL model is not renormalisable and must be regularised. Schwinger's Proper Time Regularisation (PTR) is used

$$\begin{aligned} \frac{1}{A^n} &= \frac{1}{(n-1)!}\int_0^\infty d\tau\, \tau^{n-1} e^{-\tau A} \\ &\longrightarrow \frac{1}{(n-1)!}\int_{1/\Lambda_{\mathrm{UV}}^2}^\infty d\tau\, \tau^{n-1} e^{-\tau A} \end{aligned}$$

regulated by a sharp cut-off

$$r_{\mathrm{UV}}(\tau) = \Theta\left(\tau - \frac{1}{\Lambda_{\mathrm{UV}}^2}\right)$$

Can include and IR cut-off which prevents quarks going on-shell .... Not needed here for deconfined quark matter

NJL Parameters

NJL Model Parameters fitted to Pion Phenomenology

Model	m_ℓ [MeV]	m_s [MeV]	M_ℓ [MeV]	M_s [MeV]	$\Lambda_{\rm UV}$ [MeV]	$G_{\rm S}$ [GeV^{-2}]	$G_{\rm D}$ [GeV^{-5}]
PS1	17.08	279.81	400^*	563^*	636.67	19.76	-
PS2	5.5^*	135.7^*	201.07	440.41	1078.9	3.17	-
HK	5.5	135.69	334.59	527.28	631.38	4.60	92.57

The proper time regularised (PTR) parameter sets used $f_\pi = 93$ MeV and $m_\pi = 140$ MeV. An asterisk ($*$) marks variables used in the fitting procedure. The three momentum regularised (TMR) parameter set HK [Masuda *et al*, (2013), Hatsuda *et al*, (1994)] used $f_\pi = 93$ MeV, $m_\pi = 138$ MeV, $m_K = 495.7$ MeV, $m_{\eta'} = 957.5$ MeV and $m_\ell = 5.5$ MeV. For the HK model, we have the additional coupling $G_{\rm D}$ which is the coupling strength of the determinant term.

$$iq^\mu f_\pi \delta_{ab} = -g_{\pi qq} \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\gamma^\mu\gamma_5 \frac{\lambda_a}{2} S(p+\frac{1}{2}q)\gamma_5\lambda_b S(p-\frac{1}{2}q)\right]$$

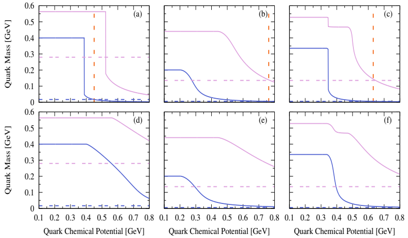
$$m_\alpha = M_\alpha - \frac{3G_{\rm S}M_\alpha}{\pi^2}\int_{\frac{1}{\Lambda_{UV}^2}}^{\infty} d\tau \frac{e^{-\tau M_\alpha{}^2}}{\tau^2} \quad ,$$

where $\alpha = \ell, s$.

At finite Density

$$
\begin{aligned}
&V_{\rm MF}^{\rm NJL}(\{M_i\},\{\mu_i\}) \\
&= 2iN_c \sum_{i\in\{u,d,s\}} \int \frac{d^4k}{(2\pi)^4} \,\mathrm{Log}\left[\frac{k^2 - M_i^2 + i\epsilon}{k^2 - M_{i0}^2 + i\epsilon}\right] \\
&\quad + \sum_{i\in\{u,d,s\}} \frac{(M_i - m_i)^2}{8G_{\rm S}} - \sum_{i\in\{u,d,s\}} \frac{(M_{i0} - m_i)^2}{8G_{\rm S}} \\
&\quad - 2N_{\rm C} \sum_{i\in\{u,d,s\}} \int \frac{d^3p}{(2\pi)^3}\, \Theta(\tilde{\mu}_i - E_{p,i})(\tilde{\mu}_i - E_{p,i}) \\
&\quad - \sum_{i\in\{u,d,s\}} \frac{(\tilde{\mu}_i - \mu_i)^2}{8G_{\rm V}} \quad ,
\end{aligned}
$$

where $\tilde{\mu}_i = \mu_i - 4G_{\rm S}\langle\psi_i^\dagger\psi_i\rangle$ and $\hat{\mu}$ =diag(μ_u,μ_d,μ_s).



The critical chemical potential to lowest order is then

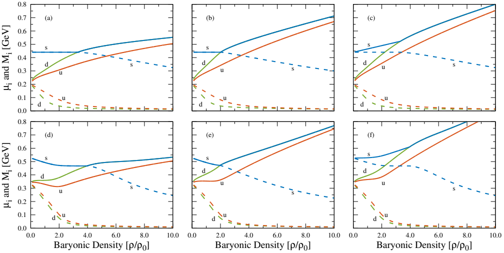
$$
\mu_{\rm crit} \simeq \frac{\Lambda_{\rm UV}}{\sqrt{2}} \simeq \begin{cases} 450 \text{ MeV} & \text{for PS1} \\ 763 \text{ MeV} & \text{for PS2} \end{cases} .
$$

Similarly, one can show in the three momentum cut-off regularisation that $\mu_{\rm crit} \simeq \Lambda_{\rm UV}$.

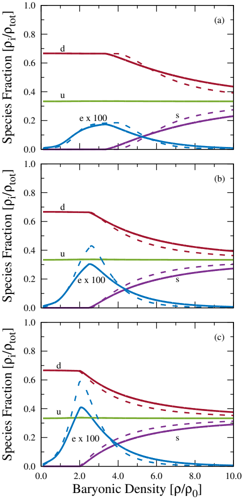
Quark Beta Equilibrium

Thermal equilibrium of quarks and leptons with respect to the weak and strong interactions

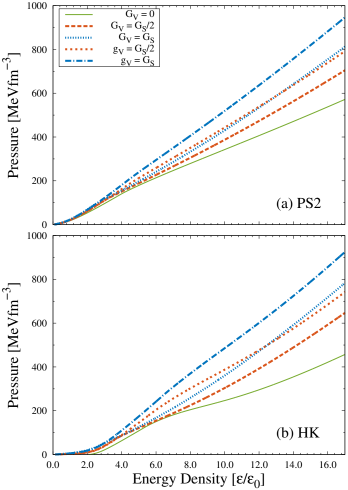
$$
\begin{aligned}
\frac{2}{3}\rho_{\mathrm{u}}^{\mathrm{v}} - \frac{1}{3}\left(\rho_{\mathrm{d}}^{\mathrm{v}} + \rho_{\mathrm{s}}^{\mathrm{v}}\right) - \rho_{\mathrm{e}}^{\mathrm{v}} - \rho_{\mu}^{\mathrm{v}} &= 0 \\
\rho - \frac{1}{3}\left(\rho_{\mathrm{u}}^{\mathrm{v}} + \rho_{\mathrm{d}}^{\mathrm{v}} + \rho_{\mathrm{s}}^{\mathrm{v}}\right) &= 0 \\
\mu_{\mathrm{d}} - \mu_{\mathrm{u}} - \mu_{\mathrm{e}} &= 0 \\
\mu_{\mathrm{d}} - \mu_{\mathrm{s}} &= 0 \\
\mu_{\mu} - \mu_{\mathrm{e}} &= 0 \quad .
\end{aligned}
$$



Quark EoS



Species Fraction [ρi/ρtot]
(a)
(b)
(c)
d
u
e x 100
s
Baryonic Density [ρ/ρ0]



GV = 0
GV = GS/2
GV = GS
gV = GS/2
gV = GS
Pressure [MeVfm−3]
(a) PS2
(b) HK
Energy Density [ε/ε0]

Why consider a crossover transition ? ?

Dissociation of hadrons does not occur naturally within models .... a phase construction is chosen and implemented using hadronic and quark models ! ! !

Typically modelled as a 1st order transition using either :

Maxwell construction

Conditions of thermal, mechanical and one component chemical equilibrium

$$\begin{aligned} T_{HP} &= T_{QP} \\ P_{HP} &= P_{QP} \\ \mu_n^{HP} &= \mu_n^{QP} \end{aligned}$$

where $\mu_n^{QP} = \mu_u + 2\mu_d$.

Local Q conserv.

Gibbs construction

Conditions of thermal, mechanical and two component chemical equilibrium

$$\begin{aligned} T_{HP} &= T_{QP} \\ P_{HP} &= P_{QP} \\ \mu_n^{HP} &= \mu_n^{QP} \\ \mu_e^{HP} &= \mu_e^{QP} \end{aligned}$$

where $\mu_n^{QP} = \mu_u + 2\mu_d$.

Global Q conserv.

- Both constructions require the EoS be reliable in the intermediate transition region BUT this may not be the case ....
  - hadron and quark models may only provide adequate description in low and high density limits, respectively
  - Quark models with no realistic confinement mechanism may have unnaturally large pressure in the transition region
- Lattice QCD at zero density and high temperature $\Longrightarrow$ crossover .... interior of phase diagram ? ? ? ?
- Extended nature of baryons suggests that the transition may occur progressively.
- INSTEAD ... assume that we understand how the low and high density matter behaves asymptotically, then parametrise our ignorance of the intermediate region where the phase transition occurs using an interpolating scheme [Masuda *et al*, (2013)].

A faux crossover transition

- Assume that we understand how the low and high density matter behaves asymptotically
- parametrise our ignorance of the intermediate region where the phase transition occurs using an interpolating scheme
- BUT .... choice of interpolating scheme is not unique
  - Masuda *et al*, 2013 P vs ρ and ϵ vs ρ hyperbolic tangent function
  - Hell and Weise P vs ϵ hyperbolic tangent function
  - Alvarez Castillo *et al*, 2014 P vs μ_B Gaussian interp.
  - Kojo *et al*, 2015 P vs μ_B polynomial interp.

One thermodynamic variable is interpolated as a function of another, then the remaining variables are calculated from the interpolated variable

Additional corrections beyond mere interpolation ....

Only a deeper understanding of QCD thermodynamics can answer which interpolation scheme is realistic and tell us the meaning behind such corrections.

Use interpolating functions :

$$f_{\pm}(\rho) = \frac{1}{2}\,(1 \pm \tanh(X)) \quad ,$$

where $X = \frac{\rho - \bar{\rho}}{\Gamma}$ and the transition region is chosen to be $\rho \in [\bar{\rho} - \Gamma, \bar{\rho} + \Gamma]$ with $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$

Interpolate Energy Density

$$\epsilon(\rho) \quad = \quad \epsilon_{\mathrm{HP}}(\rho) f_{-}(\rho) + \epsilon_{\mathrm{QP}}(\rho) f_{+}(\rho) \quad .$$

Calculate Pressure

$$P(\rho) = \rho^2 \frac{\partial(\epsilon/\rho)}{\partial\rho} \quad .$$

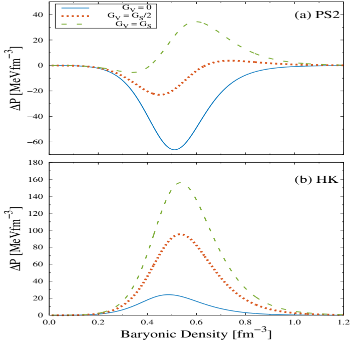
$$P(\rho) = P_{\mathrm{HP}}(\rho) f_{-}(\rho) + P_{\mathrm{QP}}(\rho) f_{+}(\rho) + \Delta P \quad ,$$

Thermodynamic correction

$$\Delta P = \rho\left[\epsilon_{\mathrm{QP}}(\rho) g_{+}(\rho) + \epsilon_{\mathrm{HP}}(\rho) g_{-}(\rho)\right] \quad .$$

with

$$g_{\pm}(\rho) = \frac{df_{\pm}(\rho)}{d\rho} = \pm\frac{2}{\Gamma}(e^{X} + e^{-X})^{-2} \quad .$$

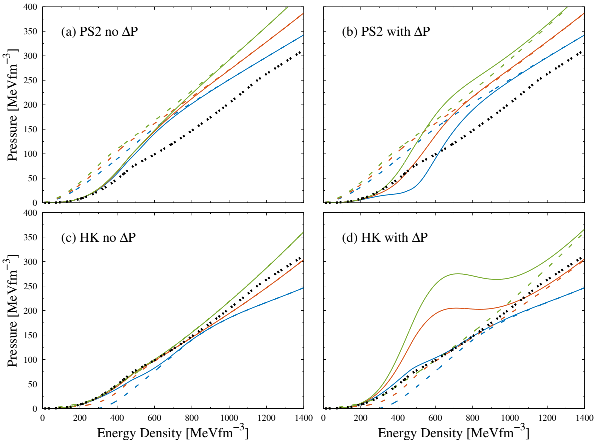


Cannot interpolate indiscrimantly !

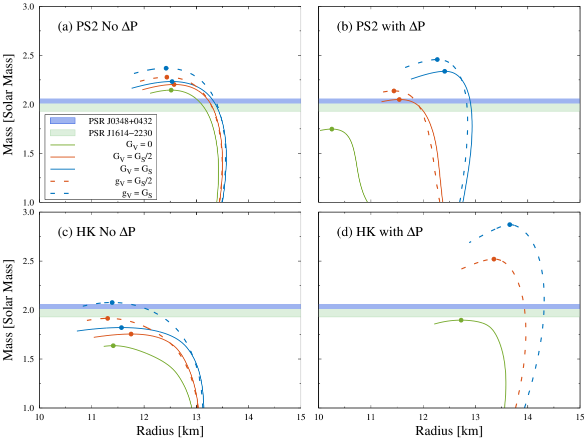
Stability : $\frac{dP}{d\rho} > 0$

Causal : $c_s^2 = \frac{dP}{d\epsilon} < 1$

Crossover results



Crossover results



Summary, in progress work and things to come

A very short summary :

- Fock terms change the delicate balance of attractive and repulsive contributions to the binding energy and tend to lead to a stiffer EoS at high density
- $\rho - N$ tensor interaction is important and should not be neglected
- Large neutron star masses 1.9–$2.1 M_{\odot}$ are obtainable with hyperons , BUT ... difficult to simultaneously describe nuclear matter properties, hyperon optical potentials and also observations of massive stars.
- Massive neutron stars can be explained assuming a cross-over transition $\rho \sim 3\rho_0$
- Many open questions about the interpolation dependence and the meaning of ΔP

Inprogress and Future Work

Further things to consider in the **QMC model** :

- Replace the bag model with other models such as NJL (See tomorrows talk).
- Investigate the effect of Fock terms and in particular tensor contributions to the properties of finite nuclei and hypernuclei.

Further things to consider in modelling **hybrid stars** :

- Non-local interactions,
- Superconducting phases,
- investigate cross-over transition with less ad-hoc construction

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The End

Thank You For Your Attention