A few very short comments on the incompressibility of SNM in the QMC model

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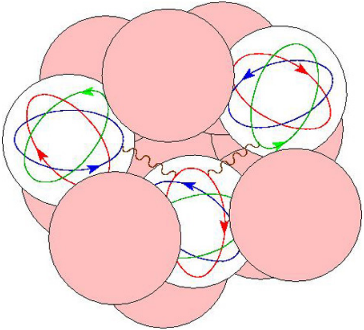
Overview



Quark-Meson Coupling Model

Quarks couple to mesons

Original idea due to Guichon (1988)

Improved and generalised to finite nuclei by Guichon, Rodinov and Thomas [Guichon *et al*, (1996)]. A lot of work has been done on developing the QMC model [Stone *et al*, (2007)].

Starts with a quark model (MIT bag) and introduces a relativistic Lagrangian with mesons coupling to quarks.

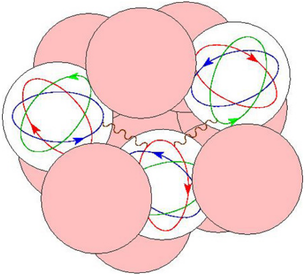


FIGURE : QMC Model (Guichon)

Meson couplings determined by saturation properties of nuclear matter :

$$g_\sigma^q,\, g_\omega^q\,,\, g_\rho^q \Longleftrightarrow \rho_0,\, \mathcal{E}_0,\, a_{\mathrm{sym}}$$

Strong Interaction

Considered through the exchange of massive mesons :

- σ scalar-isoscalar (attractive)
- ω vector-isoscalar (repulsive)
- ρ vector-isovector (isospin dependent)
- π (Only contributes through its Fock term due to parity considerations)

Only couple to the light quarks.

Could be supplemented with heavy mesons δ, K^*, σ^*, ϕ, ... of course more model dependent

Differences between our current version and the previous QMC700

In the derivation of the “relativistic" QMC700 from 2006-7 papers, additional approximations were applied :

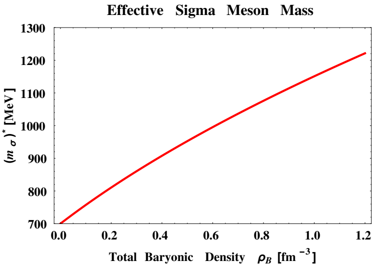
- a heavy baryon like approx. $p \ll M_{\mathrm{N}}$
  - there are extra kinematic terms
  - Integrand of the Fock terms are not just a Yukawa propagator !
- neglected the tensor interaction of vector mesons
  - Significant for the ρ meson
- Simple contact subtraction used and did not consider a form factor
- included an in-medium change in the σ meson mass calculated at the hadronic level, which only appears in the Fock term
  - all other in-medium effects on the mesons were neglected
  - at this level of approx. reasonable to neglect (or should we consider from the quark level ?)

2006-7 QMC700

$$\frac{\langle H_\sigma \rangle}{V} = \langle K(\bar\sigma)\rangle + \frac{1}{2m_\sigma^2}\left(\langle \frac{\partial K}{\partial \bar\sigma}\rangle\right)^2 + \frac{1}{(2\pi)^6}\sum_f \int d^3k_1 d^3k_2 \frac{\frac{\partial}{\partial\bar\sigma}\sqrt{k_1^2+M^{*\,2}}\,\frac{\partial}{\partial\bar\sigma}\sqrt{k_2^2+M^{*\,2}}}{(k_1-k_2)^2+m_\sigma^{*\,2}}$$

Refs. [Guichon *et al*, (2006), Stone *et al*, (2007)]

$$m_\sigma^{*\,2} = m_\sigma^2 + \langle \frac{\partial^2 K}{\partial \bar\sigma^2}\rangle \simeq m_\sigma^2(1+G_\sigma d\rho_B)$$



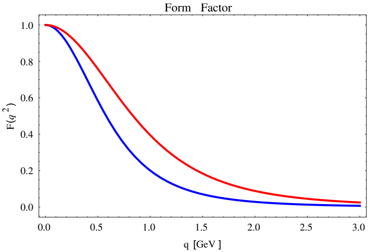
$$\frac{\langle V_\omega \rangle}{V} = \frac{G_\omega}{2}\left(\sum_f w_f^\omega n_f\right)^2 - G_\omega \sum_f \left(w_f^\omega\right)^2 \frac{1}{(2\pi)^6}\int d^3k_1 d^3k_2 \frac{m_\omega^2}{(k_1-k_2)^2+m_\omega^2}$$

$$\frac{\langle V_\rho \rangle}{V} = \frac{G_\rho}{2}\left(\sum_{mts} m n_{tms}\right)^2 - G_\rho \sum_{tmm's} \vec{I}^{\,t}_{mm'}\cdot\vec{I}^{\,t}_{m'm}\frac{1}{(2\pi)^6}\int d^3k_1 d^3k_2 \frac{m_\rho^2}{(k_1-k_2)^2+m_\rho^2}$$

and the pion ...

Consider the following :

- Pion Fock term
- Effective sigma mass in Fock term
- Form factor

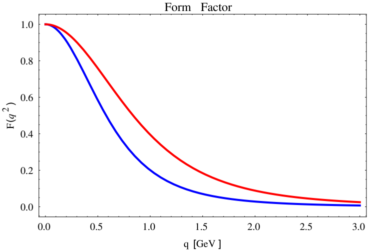


Effect on Nuclear Matter Properties

Model	B. En.* [MeV]	$S_0{}^*$ [MeV]	$\rho_0{}^*$ [fm^{-3}]	σ_0 [MeV]	$(m^*)_\sigma$ [MeV]	$(M^*)_N$ [MeV]	K_0 [MeV]	L_0 [MeV]
QMC700	-15.865	30.000	0.16000	22.789	789.56	695.21	340.42	53.270
m_σ	-15.865	30.000	0.16000	22.716	700.00	697.14	315.39	52.413
m_σ^*,π	-15.865	30.000	0.16000	22.385	784.38	705.67	320.77	52.138
m_σ,π	-15.865	30.000	0.16000	22.287	700.00	708.15	298.14	51.711
m_σ^*,π,$\Lambda = 0.9$	-15.865	30.000	0.16000	21.605	775.38	724.71	319.25	67.870
m_σ,π,$\Lambda = 0.9$	-15.865	30.000	0.16000	21.617	700.00	724.43	306.02	66.191
m_σ^*,π,$\Lambda = 1.3$	-15.865	30.000	0.16000	21.898	778.61	717.74	322.25	63.099
m_σ,π,$\Lambda = 1.3$	-15.865	30.000	0.16000	21.877	700.00	718.23	305.08	61.407
m_σ,π,$\Lambda_{\omega\rho\pi} = 0.9$	-15.865	30.000	0.16000	21.973	700.00	715.91	286.87	56.786

Consider the following :

- Pion Fock term
- Effective sigma mass in Fock term
- Form factor



Fock Energy Densities

Model	B. En.* [MeV]	S_0* [MeV]	ρ_0* [fm^{-3}]	$(\epsilon^F)_\sigma$ [MeVfm^{-3}]	$(\epsilon^F)_\omega$ [MeVfm^{-3}]	$(\epsilon^F)_\rho$ [MeVfm^{-3}]	$(\epsilon^F)_\pi$ [MeVfm^{-3}]
QMC700	-15.865	30.000	0.16000	2.8904	-4.0527	-1.8919	0
m_σ	-15.865	30.000	0.16000	3.5514	-3.6968	-1.4922	0
m_σ^*,π	-15.865	30.000	0.16000	2.8218	-3.9634	-1.6408	-0.89496
m_σ,π	-15.865	30.000	0.16000	3.4185	-3.6198	-1.2709	-0.89496
m_σ^*,π,$\Lambda = 0.9$	-15.865	30.000	0.16000	1.9162	-2.2620	-0.89707	-0.70842
m_σ,π,$\Lambda = 0.9$	-15.865	30.000	0.16000	2.3041	-2.1782	-0.81086	-0.70842
m_σ^*,π,$\Lambda = 1.3$	-15.865	30.000	0.16000	2.3018	-2.8992	-1.1361	-0.79400
m_σ,π,$\Lambda = 1.3$	-15.865	30.000	0.16000	2.7760	-2.7420	-0.97054	-0.79400
m_σ,π,$\Lambda_{\omega\rho\pi} = 0.9$	-15.865	30.000	0.16000	3.3228	-2.0288	-0.21545	-0.70842

The Devil is in the Details

The Hadronic Lagrangian

The QMC Lagrangian density used in this work is given by a combination of baryon, meson, and lepton components

$$\mathcal{L} = \sum_B \mathcal{L}_B + \sum_m \mathcal{L}_m + \sum_\ell \mathcal{L}_\ell \;,$$

for the octet of baryons $B \in \{N, \Lambda, \Sigma, \Xi\}$, selected mesons $m \in \{\sigma, \omega, \rho, \pi\}$, and leptons $\ell \in \{e^-, \mu^-\}$ densities.

$$\begin{aligned}
\Gamma_{\sigma B} &= g_{\sigma B} C_B(\bar{\sigma}) F^{\sigma}(k^2)\mathbf{1} \\
&= -\frac{\partial M_B^*}{\partial \bar{\sigma}} F^{\sigma}(k^2)\mathbf{1} \;, \\
\mathbf{\Gamma}_{\eta B} &= \epsilon_\eta^\mu \mathbf{\Gamma}_{\mu\eta B} \\
&= \epsilon_\eta^\mu \left[g_{\eta B}\gamma_\mu F_1^\eta(k^2) + \frac{i f_{\eta B}\sigma_{\mu\nu}}{2M_B^*} k^\nu F_2^\eta(k^2) \right] \boldsymbol{t} \;; \\
&\quad \eta \in \{\omega, \rho\} \;, \\
\mathbf{\Gamma}_{\pi B} &= i\frac{g_A}{2f_\pi} F^{\pi}(k^2)\gamma^\mu k_\mu \gamma_5 \boldsymbol{\tau} \;,
\end{aligned}$$

Summary of Approximations

E.O.M. are a system of coupled equations that are very difficult to solve. Need to make approximations to make the problem tractable !

- Consider nuclear matter (infinite and homogeneous, No Coulomb interaction nor Surface effects)
- No Dirac sea
- Mean Field Approximation (Hartree or Hartree-Fock Approximation)
- Zero temperature
- Time independence (Static Approximation)
- Form factors (Dipole) and contact subtraction ($\delta(\vec{r}) \mapsto \xi \times \delta(\vec{r})$, $\xi = 0$) in Fock terms

Current Version with Extra Physics

Current Version with Tensor interaction

Model	B. En.* [MeV]	S_0* [MeV]	ρ_0* [fm^{-3}]	Λ [MeV]	R_N [fm]	g_σ	g_ω	g_ρ	K_0 [MeV]	L_0 [MeV]
Dirac Only	-15.865	32.5	0.16	900	1.	10.1	9.22	7.84	294	85
	-15.865	30	0.16	900	0.8	10.51	9.78	7.44	301	79
	-15.865	32.5	0.16	1300	1.	10.41	10.02	8.54	301	85
	-15.865	30	0.16	1300	0.8	10.88	10.68	8.34	308	79
Standard	-15.865	32.5	0.16	900	1.	8.97	9.38	4.96	273	84
	-15.865	30	0.16	900	0.8	9.57	9.86	4.64	282	78
	-15.865	32.5	0.16	1300	1.	9.31	10.67	5.4	289	88
	-15.865	30	0.16	1300	0.8	9.92	11.18	5.18	297	82

Current Version with Extra Physics

Current Version with Tensor interaction

Model	B. En.* [MeV]	S_0* [MeV]	ρ_0* [fm^{-3}]	Λ [MeV]	R_N [fm]	$(\epsilon^F)_\sigma$ [MeVfm^{-3}]	$(\epsilon^F)_\omega$ [MeVfm^{-3}]	$(\epsilon^F)_\rho$ [MeVfm^{-3}]	$(\epsilon^F)_\pi$ [MeVfm^{-3}]
Dirac Only	-15.865	32.5	0.16	900	1.	1.55	-3.1	-1.69	-0.67
	-15.865	30	0.16	900	0.8	1.7	-3.54	-1.55	-0.66
	-15.865	32.5	0.16	1300	1.	1.87	-4.33	-2.38	-0.74
	-15.865	30	0.16	1300	0.8	2.07	-5.01	-2.31	-0.74
Standard	-15.865	32.5	0.16	900	1.	1.35	-2.83	-5.03	-0.67
	-15.865	30	0.16	900	0.8	1.53	-3.18	-4.47	-0.67
	-15.865	32.5	0.16	1300	1.	1.66	-4.34	-7.06	-0.75
	-15.865	30	0.16	1300	0.8	1.88	-4.85	-6.59	-0.74

Bag Model Parametrisation

Solve self-consistently for the internal structure and parameterise as a function of the mean scalar field

$$M_B^* = M_B - w_{\sigma B} g_{\sigma N} \bar{\sigma} + \frac{d}{2} \tilde{w}_{\sigma B} \left(g_{\sigma N} \bar{\sigma}\right)^2$$

We can solve for the EoS at the hadronic level (Walecka/QHD Models).
Sub-structure of the baryons are entirely contained in the mass parameterisation.

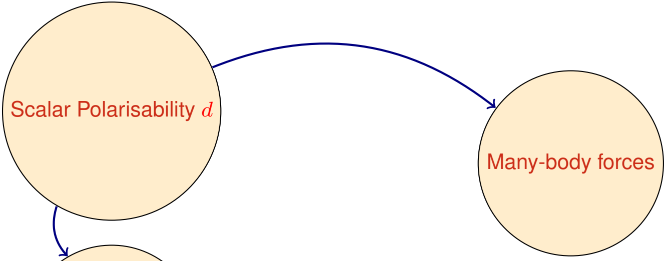
The mass parameterisation includes gluon exchange

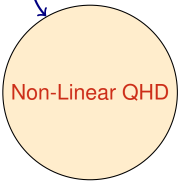
$$M(R) = \sum_i \frac{\Omega_i - Z_i}{R} + \frac{4\pi}{3} R^3 B + \Delta E_g^M$$

$$\Delta E_g^M = -\frac{\alpha_s}{2} \sum_{i<j} \int d^3x \, \vec{B}_i^a \cdot \vec{B}_j^a$$

Λ–Σ mass splitting,
Unbound Σ–hypernuclei
[Guichon *et al*, (2008)]

Scalar Polarisability d





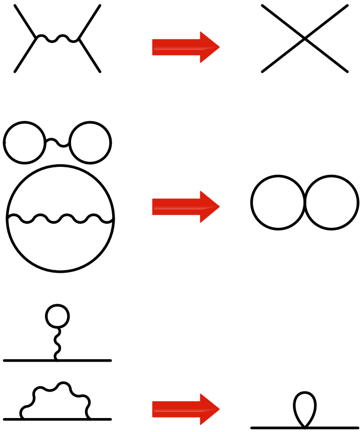
- Can re-write non-linear coupling as a linear coupling and non-linear self-coupling
- Scalar polarisability possible physical origin of NL versions of Quantum Hadro-Dynamics (QHD, Baryons pt particles)

- (Guichon and Thomas,2004) derived a Skyrme type effective force with 2, 3 and 4-body forces from the QMC model.
- Improved in (Guichon *et al*, 2006)
- d was the key ingredient, many-body terms proportional to powers of d
- Many-body interactions were a direct consequence of nucleon substructure

Nambu-Jona-Lasinio Model

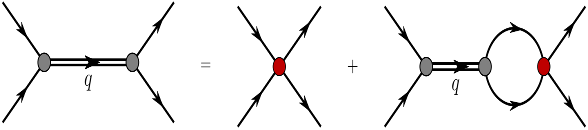
Lagrangian and Effective Potential

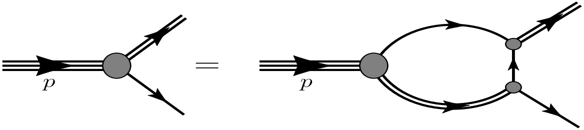
4 Fermi Contact Interaction Between Quarks



Faddeev Equations for Baryons

NJL Faddeev Eqn. see [Ishii *et al*, 1993] & [Bentz *et al*, 2001]
This is work done with Manuel Carrillo-Serrano, see his PhD thesis and [Carrillo-Serrano *et al*, 2014]





Details :

- Faddeev approximation
- Schwinger's proper regularisation
- Static approximation

Obtain :

$$M_B^* = M_B - w_B g_{\sigma N}\bar{\sigma} + \frac{d}{2}\tilde{w}_B\,(g_{\sigma N}\bar{\sigma})^2\ ,\ \text{where}\ \bar{\sigma} = \frac{\Phi}{\sqrt{Z_\pi(0)}}\quad ,\quad \Phi = M_\ell^* - M_\ell\ .$$

Work to appear on arXiv soon

Fits to obtain mass parametrisation :

$$M_B^* = M_B - w_B g_{\sigma N}\bar{\sigma} + \frac{d}{2}\tilde{w}_B \left(g_{\sigma N}\bar{\sigma}\right)^2 \ , \ \text{where } \bar{\sigma} = \frac{\Phi}{\sqrt{Z_\pi(0)}} \quad , \quad \Phi = M_\ell^* - M_\ell \, .$$

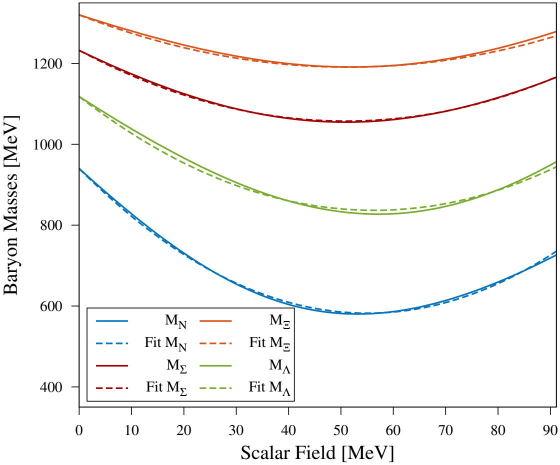
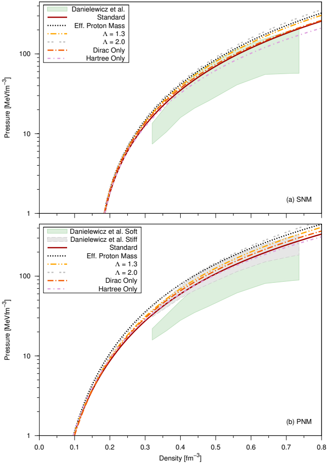
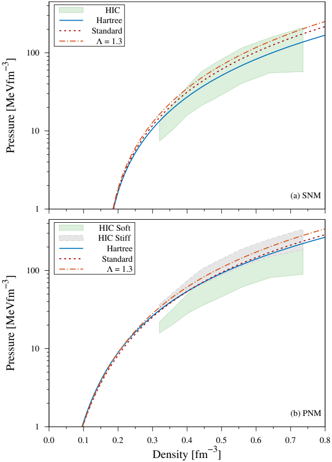


FIGURE : M. Carrillo-Serrano

Nuclear Matter Properties

Work to appear on arXiv soon





Work to appear on arXiv soon

Current Version with NJL Mass Parametrisation

Model Scenario	$g_{\sigma N}$	$g_{\omega N}$	g_{ρ}	K_0 [MeV]	L_0 [MeV]	U_{Λ} [MeV]	U_{Σ^-} [MeV]	U_{Ξ^-} [MeV]
Hartree	9.65	6.8	8.54	261	87	-55	-17	-26
Standard	8.29	8.36	4.92	263	81	-5	27	-5
$\Lambda = 1.3$	8.55	9.48	5.24	278	84	16	49	5
Dirac Only	9.41	7.95	7.66	277	82	-33	5	-19
$F_{\sigma}(\vec{k}) = 1$	8.86	8.11	4.24	259	75	-22	14	-14

Summary and Possible Future Work to Consider

A very short summary :

- Fock terms change the delicate balance of attractive and repulsive contributions to the binding energy
  - Approx. used for the Fock terms affect nuclear properties
  - As pointed out by Guichon the pion Fock decreases K_0
  - Treatment of of m_σ^* in the Fock term affects K_0
  - and so does the use of form factors
- $\rho - N$ tensor interaction is important and should not be neglected
- model of hadron structure influences nuclear observables

Further things to consider in the QMC model :

- Handle contact terms and short range correlations in a more sophisticated manner
- Extend the NJL model of baryons beyond the static approximation

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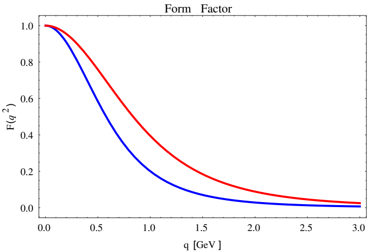
Backup Slides

Consider the following :

- Pion Fock term
- Effective sigma mass in Fock term
- Form factor



Effect on couplings

Model	B. En.* [MeV]	S_0* [MeV]	ρ_0* $[\text{fm}^{-3}]$	G_σ $[\text{fm}^{-2}]$	G_ω $[\text{fm}^{-2}]$	G_ρ $[\text{fm}^{-2}]$	g_σ	g_ω	g_ρ
QMC700	-15.865	30.000	0.16000	11.318	7.2455	4.5186	11.934	10.675	8.3572
m_σ	-15.865	30.000	0.16000	11.188	6.6092	3.5640	11.865	10.196	7.4221
m^*_σ,π	-15.865	30.000	0.16000	10.626	7.0859	3.9188	11.564	10.557	7.7828
m_σ,π	-15.865	30.000	0.16000	10.466	6.4715	3.0355	11.476	10.089	6.8498
$m^*_\sigma,\pi,\Lambda = 0.9$	-15.865	30.000	0.16000	9.4342	5.6759	3.0062	10.896	9.4486	6.8167
$m_\sigma,\pi,\Lambda = 0.9$	-15.865	30.000	0.16000	9.4511	5.4656	2.7173	10.906	9.2719	6.4809
$m^*_\sigma,\pi,\Lambda = 1.3$	-15.865	30.000	0.16000	9.8611	6.1683	3.2287	11.140	9.8500	7.0644
$m^*_\sigma,\pi,\Lambda = 1.3$	-15.865	30.000	0.16000	9.8304	5.8339	2.7582	11.122	9.5793	6.5294
$m_\sigma,\pi,\Lambda_{\omega\rho\pi} = 0.9$	-15.865	30.000	0.16000	9.9747	5.0907	0.72201	11.204	8.9484	3.3407